Starter

12 Solve these equations for $0° \leq \theta \leq 360°$

a $2\cos^2\theta + \sin\theta = 1$

b $\cos^2\theta + \cos\theta = \sin^2\theta$

c $6\sin^2\theta + 5\cos\theta = 5$

d $\tan^2\theta = 2 + \dfrac{1}{\cos\theta}$

a

$$2(1-\sin^2\theta)+\sin\theta=1$$

$$2\sin^2\theta-\sin\theta-1=0$$

$$(2\sin\theta+1)(\sin\theta-1)=0$$

$$\sin\theta=-\frac{1}{2}\text{ or }+1$$

$$\theta=180°+30°$$

$$\text{or }360°-30°$$

$$\text{or }90°$$

$$\theta=210°, 330°, 90°$$

b

$$\cos^2\theta+\cos\theta=1-\cos^2\theta$$

$$2\cos^2\theta+\cos\theta-1=0$$

$$(2\cos\theta-1)(\cos\theta+1)$$

$$\cos\theta=+\frac{1}{2}\text{ or }-1$$

$$\theta=60°$$

$$\text{or }360°-60°$$

$$\text{or }180°$$

$$\theta=60°, 300°, 180°$$

c $6(1-\cos^2\theta)+5\cos\theta=5$

$6\cos^2\theta-5\cos\theta-1=0$

$(6\cos\theta+1)(\cos\theta-1)=0$

$\cos\theta=-\dfrac{1}{6}$ or $+1$

$\theta=180°\pm 80.4°$ or $0°$ or $360°$

$\theta=99.6°, 260.4°, 0°, 360°$

d $\dfrac{\sin^2\theta}{\cos^2\theta}=2+\dfrac{1}{\cos\theta}$

$1-\cos^2\theta=2\cos^2\theta+\cos\theta$

$3\cos^2\theta+\cos\theta-1=0$

$\cos\theta=\dfrac{-1\pm\sqrt{1+12}}{6}$

$=\dfrac{1+\sqrt{13}}{6}$

$\theta=64.3°$

or $360°-64.3°$

or $180°\pm 39.8°$

$\theta=64.3°, 295.7°, 219.9°, 140.1°$

R Forces and Newton's laws

R4	Understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors); application to problems involving smooth pulleys and connected particles; resolving forces in 2 dimensions; equilibrium of a particle under coplanar forces.

Assessed at AS and A-level

Teaching guidance

Connected particles

Students should:

- understand that usually strings will be modelled as light and inextensible
- understand that usually pulleys will be modelled as light and smooth
- understand that usually pegs will be modelled as smooth.

R Forces and Newton's laws

R4

Understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors); application to problems involving smooth pulleys and connected particles; resolving forces in 2 dimensions; equilibrium of a particle under coplanar forces.

Notes

- Questions can be set involving objects that can be modelled as particles and are connected by a light, inextensible string.
- Questions can be set that involve contexts such as a car towing a trailer or several carriages connected together as a train.
- At AS, questions will be restricted to connected particles that move horizontally or vertically. Questions involving inclined planes will **not** be set.
- When particles are connected by a string so that they do not move in the same direction, the system must **not** be treated as moving with the same acceleration and the motion of each particle must be considered separately.

8.4 Systems of Forces

You can solve problems involving **connected particles** by considering the particles **separately**, or, if they are moving in the same **straight line**, as a **single particle**.

Note: Particles need to remain **in contact** or be **connected** by an **inextensible rod or string** to be considered as a **single particle**.

8.4 Systems of Forces

Example 1a

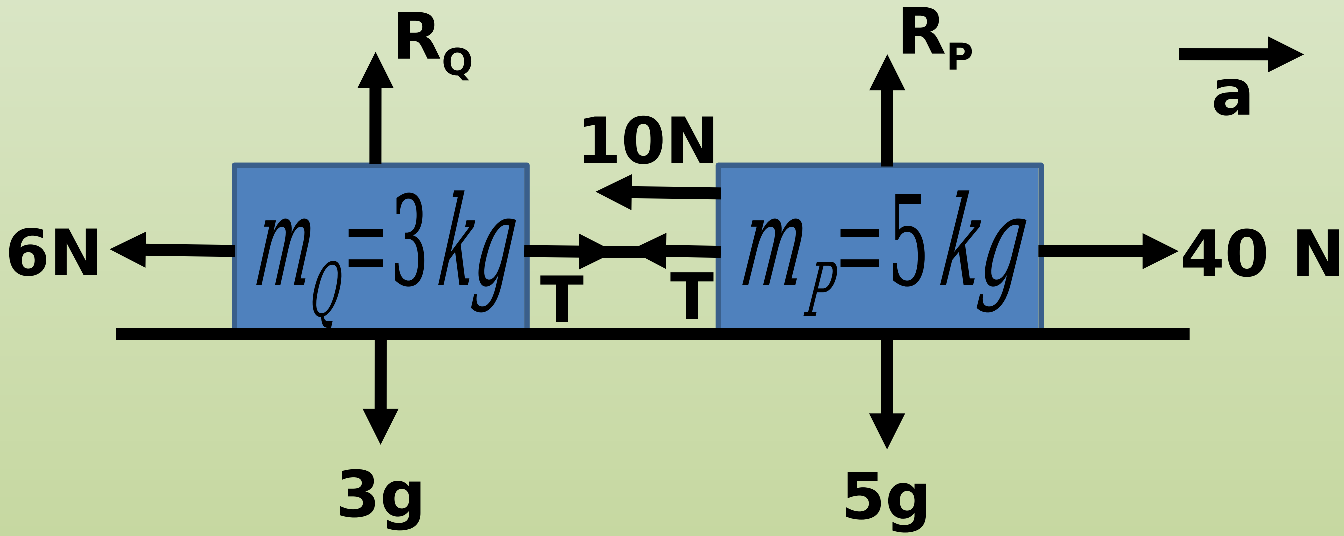
Two particles, P and Q, of masses 5 kg and 3 kg respectively, are connected by a light inextensible string. Particle P is pulled by a horizontal force of magnitude 40 N along a rough horizontal plane.

Particle P experiences a frictional force of 10 N and particle Q experiences a frictional force of 6 N.

a Find the acceleration of the particles.

b Find the tension in the string.

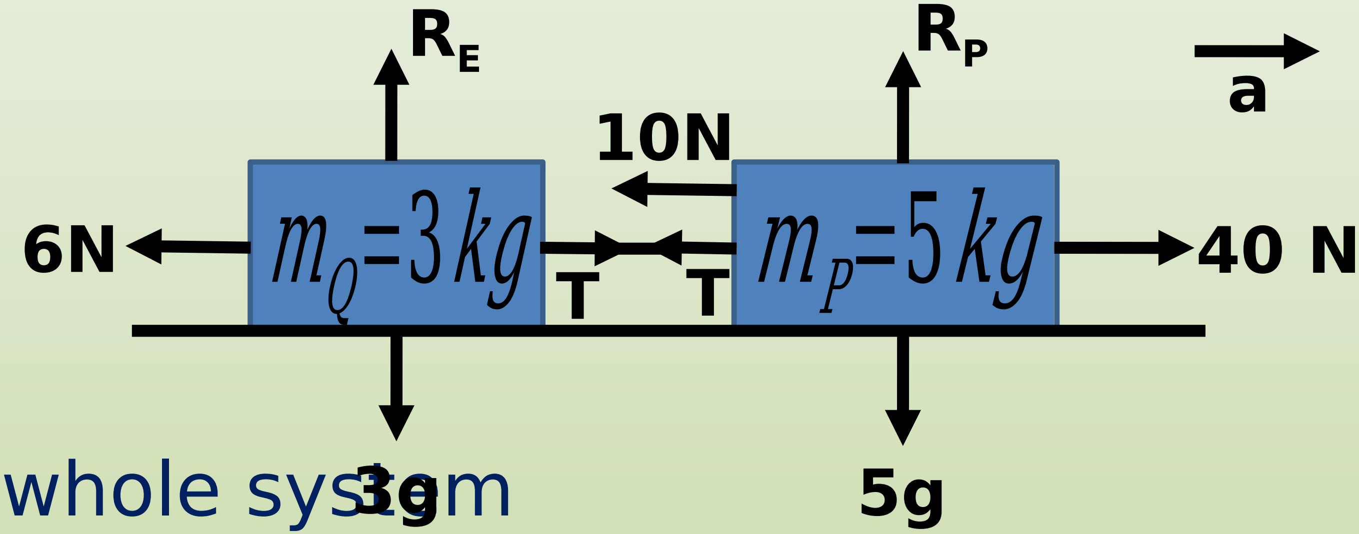
c Explain how the modelling assumptions that the string is light and inextensible have been used.



8.4 Systems of Forces

<u>Example 1a</u>

a) Find the acceleration of the particles



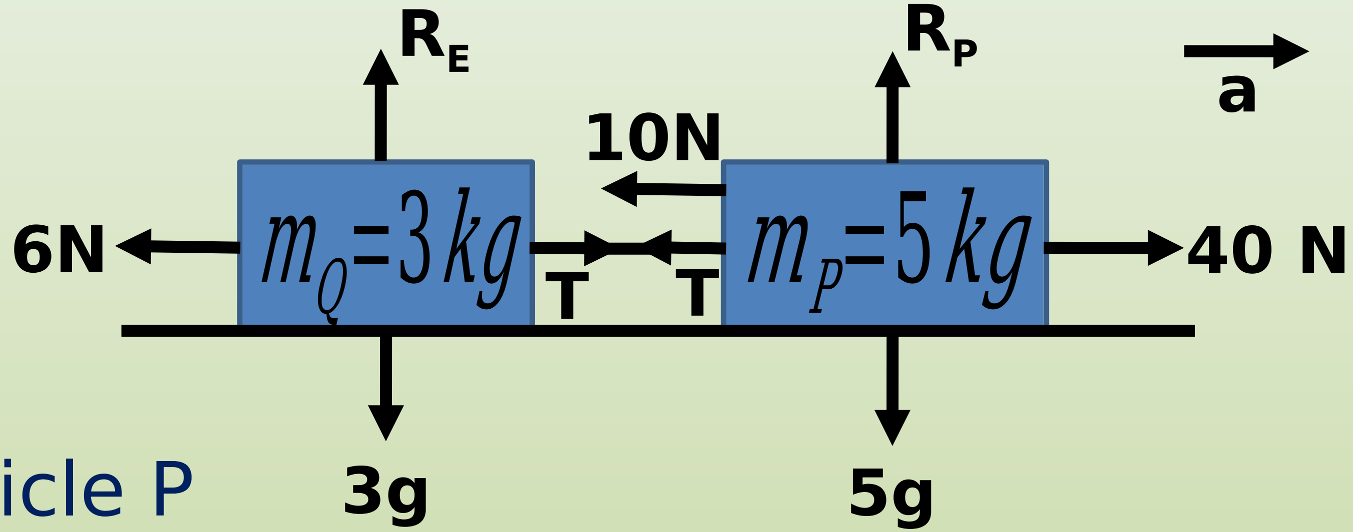
Consider the whole system

Resolve ():

8.4 Systems of Forces

Example 1b

b) Find the tension in the string



Consider particle P

Resolve ():

8.4 Systems of Forces

<u>Example 1c</u>

c) Explain how the modelling assumptions that the string is light and inextensible have been used

Inextensible □ acceleration of the masses is the same

Light □ tension is the same throughout the length of the string and the mass of the string is negligible

8.4 Systems of Forces

Example 2: you try

A car of mass 1000 kg is towing a caravan of mass 750 kg along a straight horizontal road. The caravan is connected to the car by a tow-bar which is parallel to the direction of motion of the car and the caravan. The tow-bar is modelled as a light rod. The engine of the car provides a constant driving force of 3200 N. The resistances to the motion of the car and the caravan are modelled as constant forces of magnitude 800 newtons and R newtons respectively.

Given that the acceleration of the car and the caravan is 0.88 m s^{-2},

(a) show that $R = 860$,

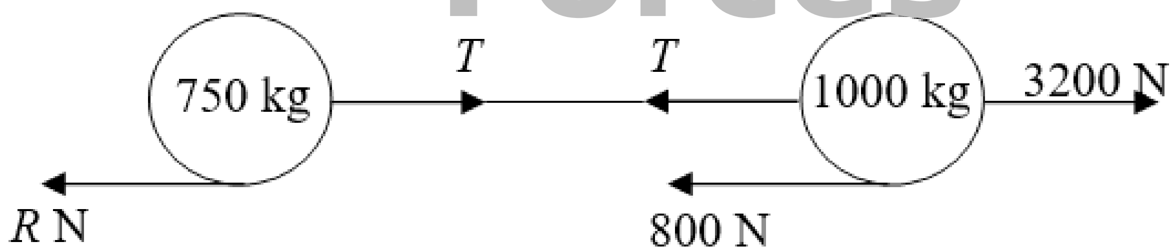
(3)

(b) find the tension in the tow-bar.

(3)

(Total 6 marks)

8.4 Systems of Forces



	For the whole system R$(\rightarrow)$ $\quad 3200 - 800 - R = 1750 \times 0.88$ Leading to $\quad R = 860$ *	 M1 A1 A1 **(3)**
	For the caravan R$(\rightarrow)$ $\quad T - 860 = 750 \times 0.88$ Leading to $\quad T = 1520 \ (\mathrm{N})$	 M1 A1 A1 **(3)**
		(6 marks)

8.4 Systems of Forces

Example 3: you try

A car of mass 1200 kg pulls a trailer of mass 400 kg along a straight horizontal road. The car and trailer are connected by a tow-rope modelled as a light inextensible rod. The engine of the car provides a constant driving force of 3200 N. The horizontal resistances of the car and the trailer are proportional to their respective masses. Given that the acceleration of the car and the trailer is 0.4 m s^{-2},

a find the resistance to motion on the trailer,

b find the tension in the tow-rope.

8.4 Systems of Forces

States, or implies in a subsequent step, that the resistances to motion will total $1600k$ (N). (Any variable is acceptable.)	**M1**
Uses $F = ma$ to write $3200 - 1600k = 1600(0.4)$	**M1**
Solves the equation to find $k = 1.6$	**A1**
Finds the resistance forces acting on the trailer: $R_{\text{trailer}} = 400 \times 1.6 = 640$ (N).	**A1**
	(4)

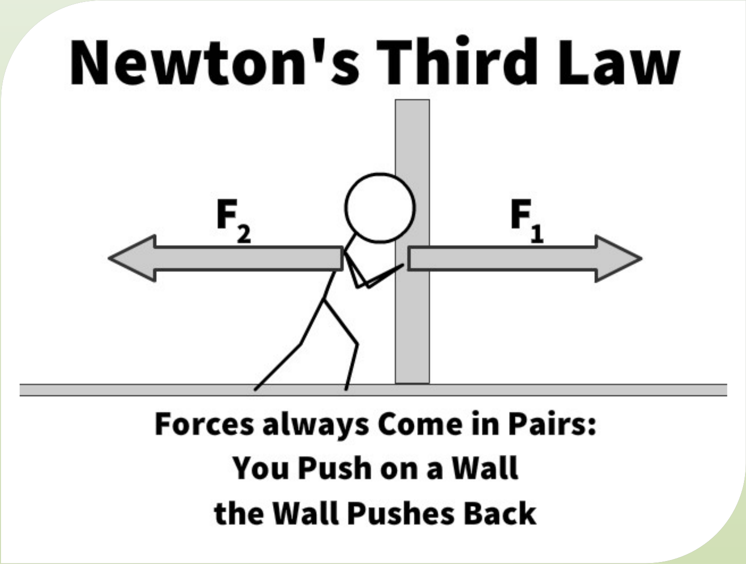
8.4 Systems of Forces

Demonstrates an understanding that the resultant force for the trailer is $T - 640$, or for the car is $3200 - 1920 - T$	**M1**
Either states $T - 640 = 400(0.4)$ using the trailer or states $3200 - 1920 - T = 1200(0.4)$ using the car.	**M1**
Correctly finds $T = 800$ (N).	**A1 ft**
	(3)

8.4 Systems of Forces

Newton’s third law states that for every action there is an equal and opposite reaction.

This means that if **two bodies are in contact**, if body A exerts a force on body B, then body B exerts a force on body A that is **equal in magnitude** and acts in the **opposite direction**

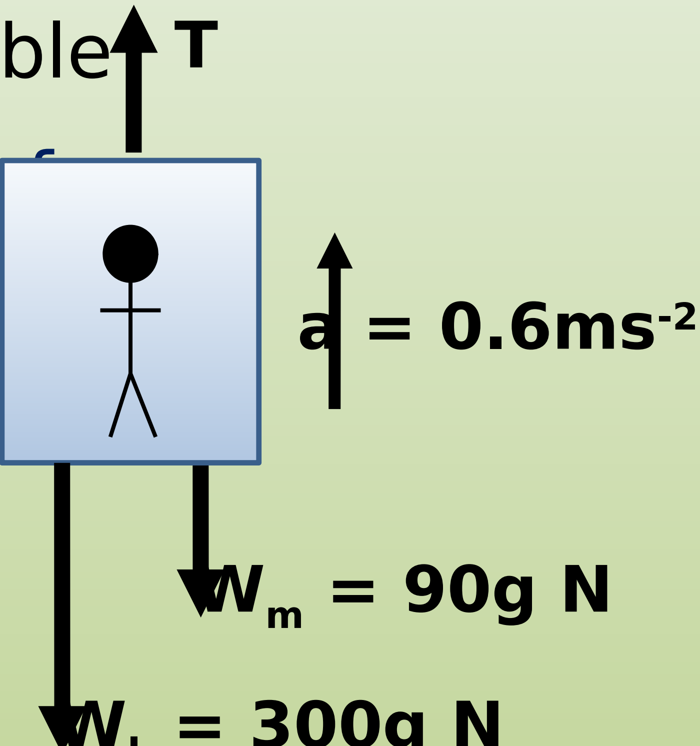


Example 4a

A man of mass 90kg is standing in a lift of mass 300kg which is accelerating upwards at $0.6ms^{-2}$.

a) Find the tension in the lift cable

Since the tension is an external f
we can consider the man and th
as a single object.



Resolve (): F=ma

$T – (W_M + W_L) = (m_L + m_M)a$

$T – (390 \times 9.8) = 390 \times 0.6$

$\therefore$ **T = 4056N**

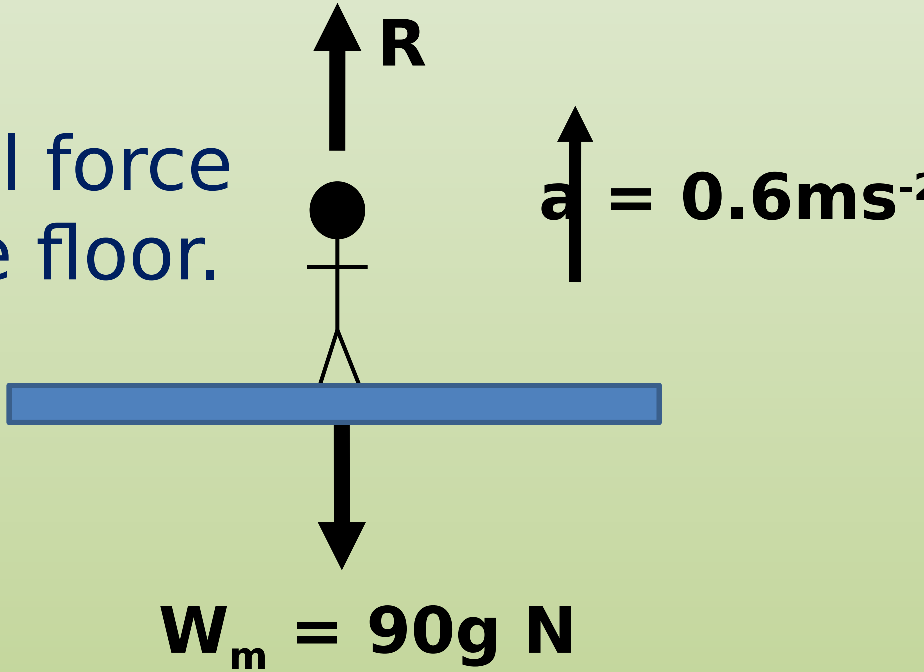
8.4 Systems of Forces

Example 4b

A man of mass 90kg is standing in a lift of mass 300kg which is accelerating upwards at 0.6ms^{-2}.

b) The reaction between the man and the floor of the lift

The reaction is an internal force between the man and the floor.



Resolve (): F=ma

R – 90g = 90 x 0.6

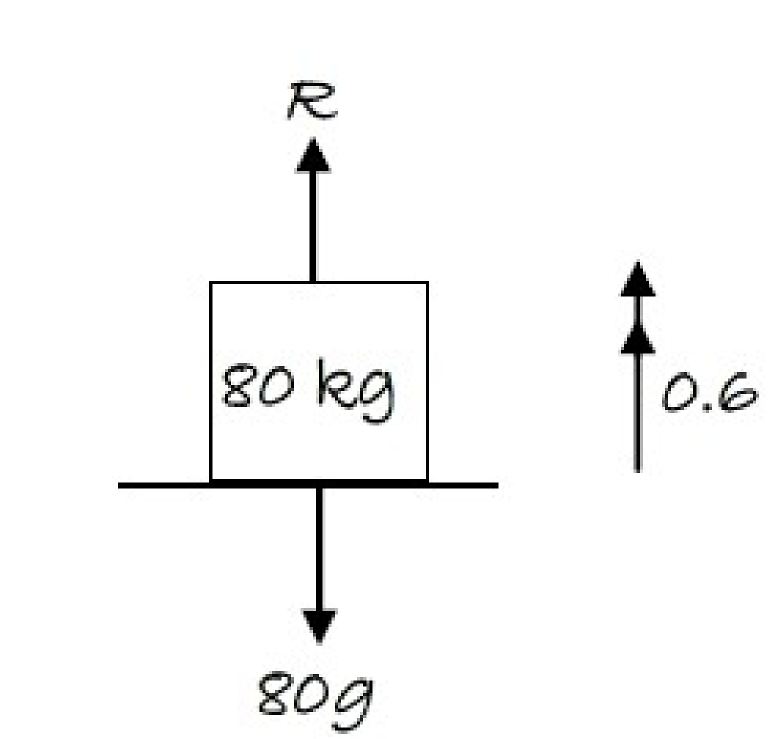
R = 54 + 90g

R = 54 + (90 x 9.8)

8.4 Systems of Forces

Example 5: You try

A man of mass 80 kg is standing in a lift, which is accelerating upwards with an acceleration of 0.6 ms^{-2}. What is the size of the force between the man and the lift floor?



$$F = ma$$

$$R - 80g = 80 \times 0.6$$

$$R = 80 \times 9.8 + 48$$

$$R = 832$$

The force is 832 N.

8.4 Systems of Forces

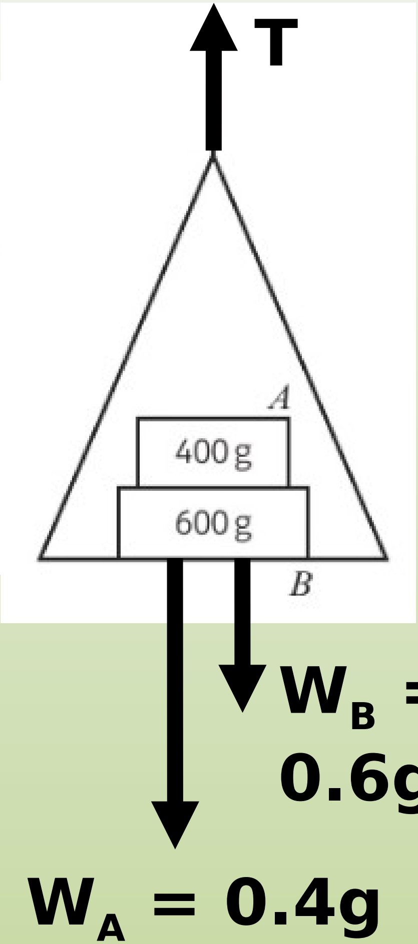
Example 6a

A light scale-pan is attached to a vertical light inextensible string. The scale-pan carries two masses A and B. The mass of A is 400 g and the mass of B is 600 g. A rests on top of B, as shown in the diagram.

The scale-pan is raised vertically, using the string, with acceleration $0.5\,\mathrm{m\,s^{-2}}$.

a Find the tension in the string.

b Find the force exerted on mass B by mass A.



Consider the whole system:

Resolve (): F=ma

$T - 0.4g - 0.6g = (0.4 + 0.6)a$

$T - g = 0.5$

∴ T = 10.3N so the tension in the string is 10N to 2sf

8.4 Systems of Forces

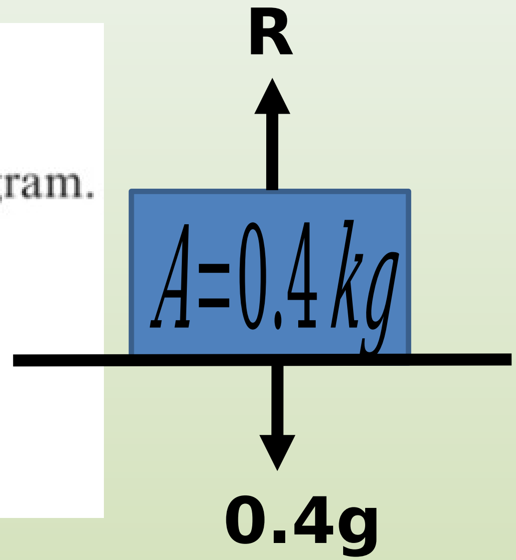
Example 6b

A light scale-pan is attached to a vertical light inextensible string. The scale-pan carries two masses *A* and *B*. The mass of A is 400 g and the mass of *B* is 600 g. *A* rests on top of *B*, as shown in the diagram.

The scale-pan is raised vertically, using the string, with acceleration 0.5 m s^{-2}.

a Find the tension in the string.

b Find the force exerted on mass *B* by mass *A*.



Resolve ():

Newton's third law states that the force exerted on mass B by mass A will be equal to the force exerted on mass A by mass B but in the opposite direction, i.e. the normal reaction on A

So the force exerted on B by A is 4.1N